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## General Certificate of Education

# Mathematics 6360

## MPC4 Pure Core 4

# Mark Scheme

*2009 examination - January series*

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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**Key to mark scheme and abbreviations used in marking**

| | |
|---|---|
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |

| | | | |
|---|---|---|---|
| √or ft or F | follow through from previous incorrect result | MC | mis-copy |
| CAO | correct answer only | MR | mis-read |
| CSO | correct solution only | RA | required accuracy |
| AWFW | anything which falls within | FW | further work |
| AWRT | anything which rounds to | ISW | ignore subsequent work |
| ACF | any correct form | FIW | from incorrect work |
| AG | answer given | BOD | given benefit of doubt |
| SC | special case | WR | work replaced by candidate |
| OE | or equivalent | FB | formulae book |
| A2,1 | 2 or 1 (or 0) accuracy marks | NOS | not on scheme |
| $-x$ EE | deduct $x$ marks for each error | G | graph |
| NMS | no method shown | c | candidate |
| PI | possibly implied | sf | significant figure(s) |
| SCA | substantially correct approach | dp | decimal place(s) |

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

**MPC4**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **1(a)** **(i)** | $f(-1)=0$ | B1 | 1 | |
| **(ii)** | $f(-\frac{1}{2})=4(-\frac{1}{8})-7(-\frac{1}{2})-3$ | M1 | | Use of $\pm\frac{1}{2}$ |
| | $=-\frac{1}{2}+\frac{7}{2}-3=0\Rightarrow$ factor | A1 | 2 | Need to see simplification ( at least $(-\frac{1}{2})^3=-\frac{1}{8}$), '=0' and conclusion |
| **(iii)** | Third factor is $(2x-3)$ | B1 | | PI |
| | $\dfrac{(x+1)(2x+1)(2x-3)}{(x+1)(2x+1)}$ | M1 | | $\dfrac{\text{3 linear factors}}{\text{2 linear factors}}$ |
| | simplifies to $2x-3$ | A1 | | Simplified result stated.<br>Alternative; see end.<br>Use remainder theorem. |
| | **Alternative**<br>Complete division to $2x+b$ | (M1) | | |
| | Complete division to $2x-3$ | (A1) | | |
| | Simplifies to $2x-3$ | (A1) | 3 | Simplified result stated |
| **(b)** | $g(-\frac{1}{2})=-\frac{1}{2}+\frac{7}{2}+d=2$ | M1 | | |
| | $d=-1$ | A1 | | |
| | **Alternative**<br>Complete division leading to rem = 2 | (M1) | | Remainder $=d+p=2$ |
| | $d=-1$ | (A1) | 2 | |
| | **Total** | | **8** | |
| **2(a)** | $R=\sqrt{10}$ | B1 | | Accept $R=3.16$ or better. |
| | $\tan\alpha=3$ | M1 | | OE (Can be implied by 71.57° seen) |
| | $\alpha=1.25$ | A1 | 3 | A0 if extra answers within given range<br>SC 1 $\tan\alpha=\frac{1}{3}$ $\alpha=0.32$ |
| **(b)(i)** | min value = $-\sqrt{10}$ ( or $\geq\sqrt{-10}$ ) | B1F | 1 | ft on $R$ |
| **(ii)** | $\sin(x-\alpha)=-1$ | M1 | | or $\sin^{-1}\frac{3\pi}{2}$ |
| | $x=5.96$ | A1F | 2 | ft on their $\alpha$ (to 2 dp) $+\frac{3\pi}{2}$ |
| | **Total** | | **6** | |

**MPC4 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **3(a)** | | | | |
| **(i)** | $\frac{2x+7}{x+2} = 2 + \frac{3}{x+2}$ | B1<br>B1 | 2 | |
| **(ii)** | $\int \frac{2x+7}{x+2} = 3\ln(x+2) + 2x + C$ | B1F<br>B1F | 2 | Either term correct<br>Both correct; constant required; condone missing bracket<br>ft on $A$, $B$ |
| **(b)(i)** | $28 + 4x^2 =$<br>$P(5-x)^2 + Q(1+3x)(5-x)$<br>$+ R(1+3x)$ | M1 | | |
| | $x = 5 \qquad x = -\frac{1}{3}$ | m1 | | Two values of $x$ used to find $R$ and $P$. |
| | $R = 8 \qquad P = 1$ | A1 | | SC $R = 8$, $P = 1$ NMS can score B1,B1 |
| | $x = 0 \Rightarrow 28 = 25P + 5Q + R$ | m1 | | Third value of $x$ used to find $Q$ |
| | $Q = -1$ | A1 | | . |
| | **Alternative**<br>$28 + 4x^2 =$<br>$P(5-x)^2 + Q(1-3x)(5-x)$<br>$+ R(1+3x)$ | (M1) | | |
| | $= (25P + 5Q + R) +$<br>$(-10P + 14Q + 3R)x + (P - 3Q)x^2$ | (m1) | | Collect terms and form equations |
| | $P - 3Q = 4$<br>$14Q + 3R - 10P = 0$<br>$25P + 5Q + R = 28$ | (A1) | | Correct equations |
| | $P = 1 \quad Q = -1 \quad R = 8$ | (m1)<br>(A1) | 5 | Solve for $P$ $Q$ and $R$ |
| **(ii)** | $\int \frac{1}{1+3x} - \frac{1}{5-x} + \frac{8}{(5-x)^2}\,dx$ | M1 | | Use partial fractions |
| | $= \frac{1}{3}\ln(1+3x) + \ln(5-x) + \frac{8}{5-x} + (C)$ | m1<br>A1F<br>A1F | 4 | $a\ln(1+3x) + b\ln(5-x)$<br>OE; both ln integrals correct; needs ( )<br>Other term correct<br>ft on their $P$, $Q$, $R$<br><br>SC: If no $P$,$Q$, $R$ found in (b)(i), can gain method marks by inserting other values or retaining the letters (max 2/4) |
| | **Total** | | **13** | |

**MPC4 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **4(a) (i)** | $(1-x)^{\frac{1}{2}}=1+\frac{1}{2}(-x)+px^2$ | M1 | | |
| | $=1-\frac{1}{2}x-\frac{1}{8}x^2$ | A1 | 2 | |
| **(ii)** | $\sqrt{4-x}=2\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$ | B1 | | or $(4)^{\frac{1}{2}}\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$ |
| | $=(2)\left(1-\frac{1}{2}\left(\frac{x}{4}\right)-\frac{1}{8}\left(\frac{x}{4}\right)^2\right)$ | M1 | | $x$ replaced by $\frac{x}{4}$; condone missing ( ) |
| | | | | Or start again with $\left(1-\frac{x}{4}\right)^{\frac{1}{2}}$ |
| | $=2-\frac{x}{4}-\frac{x^2}{64}$ | A1 | | CAO or decimal equivalent |
| | **Alternative** | | | |
| | $(4-x)^{\frac{1}{2}}=4^{\frac{1}{2}}+\frac{1}{2}\times4^{-\frac{1}{2}}(-x)$ | (M1) | | Use of $(a+x)^n$ from formula book |
| | $+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2}4^{-\frac{3}{2}}(-x)^2$ | (A1) | | Condone missing brackets and 1 error |
| | $=2-\frac{x}{4}-\frac{x^2}{64}$ | (A1) | 3 | |
| **(b)** | $x=1 \quad \sqrt{3}\approx2-\frac{1}{4}-\frac{1}{64}$ | M1 | | $x=1$ used in their expansion |
| | $=1.734$ (3dp) | A1 | 2 | CSO |
| | **Total** | | **7** | |
| **5(a)** | $\sin2x=2\sin x\cos x$ | B1 | 1 | OE, eg $\sin x\cos x+\sin x\cos x$ etc |
| **(b)** | $\cos x=0 \quad x=90,\ 270$ | B1 | | Both required |
| | $10\sin x+3=0$ | M1 | | |
| | $x=197.5 \quad 342.5$ | A1A1 | 4 | CAO<br>if extra values in given range, max 1/2 |
| **(c)** | $\cos2x=\cos^2x-\sin^2x$ | B1 | | $\cos2x$ in any correct form |
| | $2\sin x\cos x+1-2\sin^2x=1+\sin x$ | M1 | | $\sin 2x$ expanded and $\cos2x$ in terms of $\sin x$ used |
| | | A1 | | |
| | $2\sin x(\cos x-\sin x)=\sin x$ | | | |
| | $2(\cos x-\sin x)=1$ | A1 | 4 | CSO; need to see $\sin x$ taken out as factor or cancelled |
| | **Total** | | **9** | |

**MPC4 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| 6 (a) | $x^2\frac{dy}{dx}+2xy$ | M1 A1 | | Product rule used. Allow 1 error |
| | $+3y^2\frac{dy}{dx}$ | B1 | | Chain rule |
| | $=2$ | B1 | | RHS and equation with no spurious $\frac{dy}{dx}$ unless recovered. |
| | $(2, 1)$, $4\frac{dy}{dx}+4+3\frac{dy}{dx}=2$ | M1 | | Substitute $(2, 1)$ |
| | $\frac{dy}{dx}=-\frac{2}{7}$ | A1 | 6 | CSO |
| (b) | $\frac{dy}{dx}=0\Rightarrow$ | M1 | | Derivative = 0 used |
| | $xy=1$ | A1 | | OE |
| | $x^2\times\frac{1}{x}+\frac{1}{x^3}=2x+1$ | m1 | | Use $xy=k$ to eliminate $y$ on LHS |
| | $\frac{1}{x^3}=x+1$ | A1 | 4 | Answer given; CSO |
| | **Total** | | **10** | |
| 7(a) (i) | $\int\frac{dx}{e^{\frac{1}{2}x}}=\int -kt\ dt$ | B1 | | Separate; condone missing integral signs |
| | $-2e^{-\frac{1}{2}x}=-k\frac{t^2}{2}\ \ (+C)$ | B1B1 | 3 | |
| (ii) | $-2e^{-\frac{1}{2}x}=-k\frac{t^2}{2}-2e^{-3}$ | M1 | | Use $(6,0)$ to find constant |
| | $\ln\left(e^{-\frac{1}{2}x}\right)=\ln\left(k\frac{t^2}{4}+e^{-3}\right)$ | M1 | | Take logarithms correctly; condone one side negative. Must have a constant. |
| | $-\frac{1}{2}x=\ln\left(k\frac{t^2}{4}+e^{-3}\right)$ | | | |
| | $x=-2\ln\left(\frac{kt^2}{4}+e^{-3}\right)$ | A1 | 3 | Answer given; CSO |
| (b) (i) | $t=10$ $\quad x=-2\ln\left(\frac{0.004\times10^2}{4}+e^{-3}\right)$ | M1 | | |
| | $=3.8\ \Rightarrow 3800$ | A1 | 2 | CAO |
| (ii) | $x=0$ $\quad \frac{0.004\times t^2}{4}+e^{-3}=1$ | M1 | | |
| | $t=30.8$ | A1 | 2 | CAO<br>Treat 0.04 or 0.0004 as misread (–1) |
| | **Total** | | **10** | |

**MPC4 (cont)**

| Q | Solution | Marks | Total | Comments |
|---|---|---|---|---|
| **8(a)** **(i)** | $\overrightarrow{AB} = \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ | M1<br>A1 | 2 | $\pm\left( \overrightarrow{OA} - \overrightarrow{OB} \right)$<br>A0 if answer as coordinates |
| **(ii)** | $\overrightarrow{OB} \bullet \overrightarrow{AB} = 3\times1 + 1\times0 + (-2)\times(-1) = 5$ | M1<br>A1 | | Evaluate to single value |
| | $\cos\theta = \dfrac{\overrightarrow{OB} \bullet \overrightarrow{AB}}{\lvert \overrightarrow{OB} \rvert \times \lvert \overrightarrow{AB} \rvert}$<br>$\lvert \overrightarrow{OB} \rvert = \sqrt{14} \quad \lvert \overrightarrow{AB} \rvert = \sqrt{2}$ | M1 | | Use formula for $\cos\theta$ with any 2 vectors and at least one of the corresponding modulii 'correct' |
| | $\cos\theta = \dfrac{5}{\sqrt{7\times2}\sqrt{2}} = \dfrac{5}{2\sqrt{7}}$ | A1 | | CSO; AG so need to see intermediate step eg $\dfrac{5}{\sqrt{7\times2}\sqrt{2}}$ or $\dfrac{5}{\sqrt{28}}$ |
| | **Alternative**<br>cos rule attempted with cos B<br>cos rule correct with cos B<br>derive correct given form | <br>(M1)<br>(A1)<br>(A2) | 4 | |
| **(b)** | $\mathbf{r} = \begin{bmatrix} 6 \\ 2 \\ -4 \end{bmatrix} + \lambda \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ | M1<br>A1F | 2 | $\overrightarrow{OC} + \lambda\overrightarrow{AB}$. Allow one slip<br>ft on $\overrightarrow{AB}$; needs $\mathbf{r}$ or $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ |
| **(c)** | $\overrightarrow{OD} \bullet \overrightarrow{AB} = \begin{bmatrix} 6+\lambda \\ 2 \\ -4-\lambda \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ | M1 | | |
| | $6 + \lambda + 4 + \lambda = 0$ | m1 | | |
| | $\lambda = -5$ | A1F | | ft on equation of line |
| | $D$ is $(1,2,1)$ | A1 | | CAO |
| | **Alternative**<br>$\begin{bmatrix} a \\ b \\ c \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = a - c = 0$ | (M1) | | Let $D$ be $(a,b,c)$<br>Scalar product evaluated and equated to 0 |
| | $a = 6+\lambda, \quad b = 2, \quad c = -4-\lambda$ | (m1)<br>(A1) | | Use equation of line |
| | $a + c = 2$<br>$a = 1 \quad b = 2 \quad c = 1$ | (A1) | 4 | |
| | **Total** | | **12** | |
| | **TOTAL** | | **75** | |